An LPI Numerical Implicit Solution for Unsteady Mixed-Flow Simulation*

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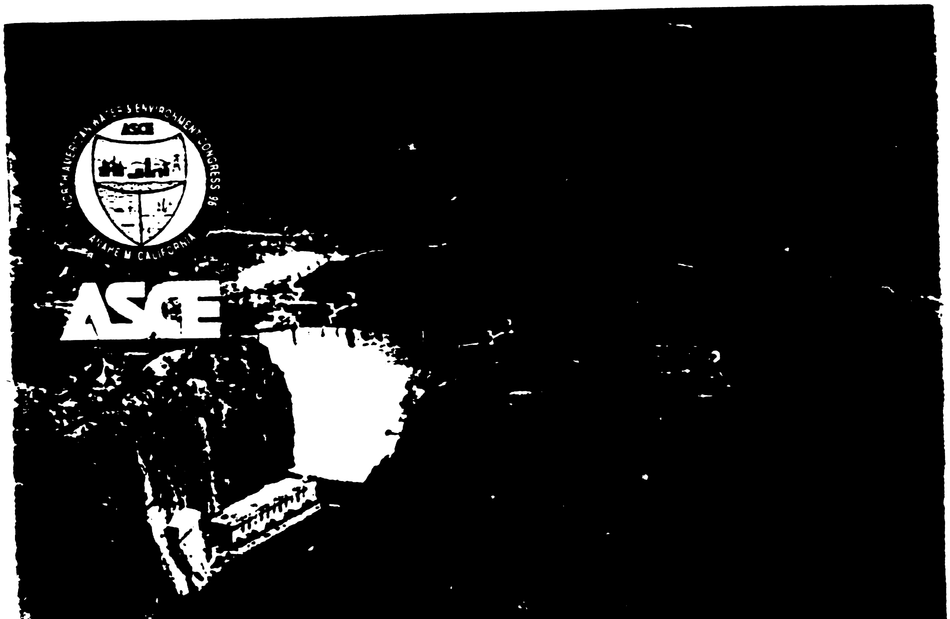

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An LPI Numerical Implicit Solution for Unsteady Mixed-Flow Simulation*

D. L. Fread[1], Ming Jin[2], Janice M. Lewis[3]

Abstract

A Local Partial Inertia (LPI) technique is developed as an option within the National Weather Service (NWS) FLDWAV dynamic flood routing model to enhance its capability to model unsteady flows with subcritical/supercritical mixed-flow regimes and moving interfaces. By neglecting varying portions of the inertial terms in the unsteady flow momentum equation according to the local Froude number, the LPI technique retains the essential accuracy associated with dynamic routing models and provides stable numerical solutions for mixed flows for the four-point implicit numerical scheme used in the FLDWAV model.

Introduction

The NWS FLDWAV model is a generalized flood routing model which is based on an implicit weighted four-point, nonlinear, finite-difference solution of the one-dimensional unsteady flow (Saint-Venant) equations. FLDWAV combines the capabilities of the popular NWS DAMBRK and DWOPER models (Fread,1993) and provides additional features, such as a Kalman filter for updating the flood forecast using the observed real-time stages, a multiple-reach routing algorithm which enables application of different routing techniques (implicit, explicit, level- pool, Muskingum-Cunge, etc.) to specified subreaches, and a new network solution algorithm for any dendritic river system. A new optional feature of FLDWAV, presented herein, uses a Local Partial Inertial (LPI) solution technique for modeling "mixed" (subcritical and/or supercritical) flows in specified subreaches.

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When modeling unsteady flows, the dynamic routing technique using the four-point implicit numerical scheme tends to be less numerically stable than the diffusion (zero inertia) routing technique for certain mixed flows, especially in the near critical range of the Froude number (F_r) or mixed flows with moving supercritical/ subcritical interfaces. It has been observed that the diffusion technique which eliminates the two inertial terms in the momentum equation produces stable numerical solutions for flows where F_r is in the range of critical flow (F_r=1.0). To take advantage of the diffusion method's stability and retain the accuracy of the dynamic method, a LPI technique is introduced in which a numerical filter (σ) modifies the extent of contribution of the inertial terms in the momentum equation such that its properties vary from dynamic to diffusion. This paper presents the LPI technique, its stability/error properties, and an application example.

Model Formulation and Error Analysis

The Saint-Venant unsteady flow equations are (Fread, 1993):

$$\partial Q/\partial x + \partial(A + A_0)/\partial t - q = 0 \tag{1}$$

$$\partial Q/\partial t + \partial(\beta Q^2/A)/\partial x + gA(\partial h/\partial x + S_f + S_e) + L + W_f B = 0 \tag{2}$$

in which t is time, x is distance along the longitudinal axis of the waterway, h is the water surface elevation, A is the active cross-sectional area of flow, A_0 is the inactive (off-channel storage) cross-sectional area of flow, q is the lateral inflow or outflow, β is the coefficient for nonuniform velocity distribution within the cross section, g is the gravity constant, S_f is the friction slope, S_e is the slope due to local expansion-contraction (large eddy loss), L is the momentum effect of lateral flow, W_f is the wind term, B is the channel flow width.

In the LPI technique, the momentum equation, Eq. (2), is modified by a numerical filter, σ, so that the inertial terms are partially or totally omitted in some situations. The modified equation and numerical filter are:

$$\sigma[\partial Q/\partial t + \partial(\beta Q^2/A)/\partial x] + gA(\partial h/\partial x + S_f + S_e) + L + W_f B = 0 \tag{3}$$

$$\sigma = \begin{cases} 1.0 - F_r^{\,m} & (F_r \leq 1.0;\quad m \geq 1) \\ 0 & (F_r > 1.0) \end{cases} \tag{4}$$

in which m is a user specified constant. Figure 1 shows the variation of σ with F_r and with the value m. The σ filter, which depends on F_r, has a variation that ranges from a linear function to the Dirac delta function. Since the Froude number is determined at each computational point for each time, σ is a "local" parameter. Therefore, portions of the routing reach with low Froude numbers will be modeled with all or essentially all of the inertial terms included, while those portions with F_r values in the vicinity of critical flow will be modeled with "partial inertial" effects included; and supercritical flows (F_r > 1) will be modeled with no inertial effects.

It is found that smaller values of m tend to stabilize the solution in some cases while larger values of m provide more accuracy. By using the σ filter, the FLDWAV model automatically changes from a dynamic model to a diffusion model and takes advantage of the stability of the diffusion model for those flows with F_r near the critical value of 1.0. Previously, a simple inertial filter ($1-Fr^2$) was proposed (Havnø and Brorsen, 1986), but it was not "localized" nor its error properties extensively analyzed.

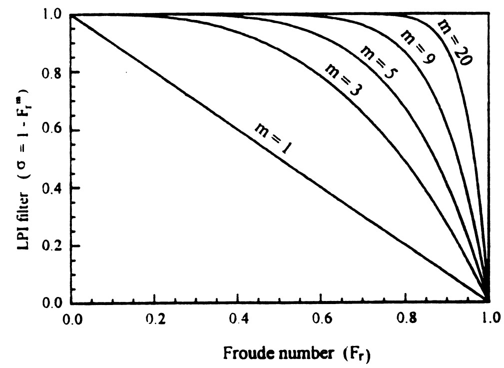

Figure 1 The LPI Filter (σ)

In order to generally evaluate the effects of the LPI technique on the accuracy of the computational results, the proportional contribution of the inertial terms to the total momentum equation is theoretically analyzed for a rectangular channel situation. Also, assuming A_0, S_e, q, L, W_f negligible, and $\beta = 1.0$, the Saint-Venant equations, Eqs (1) and (2), can be re-written in the following form:

$$\partial y/\partial t + 1/B \, \partial(AV)/\partial x = 0 \tag{5}$$

$$\left[\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right] / \left[g\left(\frac{\partial y}{\partial x} - S_0\right)\right] + gS_f / \left[g\left(\frac{\partial y}{\partial x} - S_0\right)\right] + 1.0 = 0 \tag{6}$$

in which V is the average cross-sectional flow velocity, y is the flow depth, S_0 is the channel bottom slope. The first term on the left-hand-side of Eq. (6), noted as IT in the following analysis, is the proportional contribution of the inertial terms within the momentum equation, compared with the water surface slope; therefore, IT is an indicator of the importance of the inertial terms. Using Eq. (5), the first term (IT) in Eq. (6) can be reformulated as:

$$IT = \left[\frac{\partial V}{\partial t} - \frac{V}{y}\left(\frac{\partial y}{\partial t} + V\frac{\partial y}{\partial x}\right)\right] / \left[g\left(\frac{\partial y}{\partial x} - S_0\right)\right] \tag{7}$$

To determine the most influential factors affecting the magnitude of the inertial terms, the velocity (V) is expressed by Manning's equation and the ∂y/∂x term is approximated by the kinematic assumption $\partial y/\partial x = (-1/c)\partial y/\partial t$ in which c is the kinematic wave speed evaluated by c=KV; K is a cross-sectional shape factor with a value of 1.5 for a typical river channel. Thus, Eq. (7) can be reconstructed as:

$$IT = -0.5 F_r^2 / (1 + 1.5 \phi F_r^3) \tag{8}$$

$$\phi = (n^2 g^{3/2} y^{1/6}) / (\lambda^2 \partial y/\partial t) \tag{9}$$

in which φ is a new dimensionless parameter, n is the Manning's resistance coefficient, λ is the constant in Manning's equation (λ=1.49 for English system of units and λ=1.0 for SI units). The new parameter (φ), reflects the flow's unsteadiness and hydraulic condition. The value of ϕ is found to range between 5 and 5000 for the practical spectrum of unsteady flows. Values of ϕ less than about 10 occur only when a very large unsteady flow, resulting from a nearly instantaneous failure of a high dam, propagates in a channel having a flat bed slope. Equation (8) shows that the proportional contribution of the inertial terms to the total momentum equation depends on both F_r and ϕ. Figure 2 shows IT as function of F_r for different ϕ values. According to Eq. (6), the contribution of the inertial terms in the momentum equation is negligible if the absolute value of IT approaches zero, which means that the surface slope and friction slope are

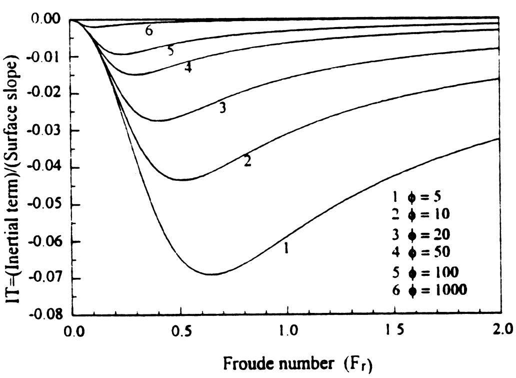

Figure 2 IT as function of F_r and ϕ

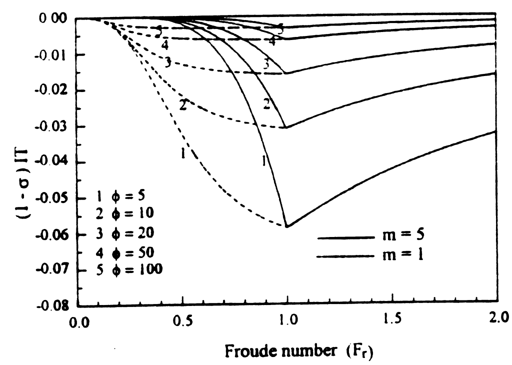

Figure 3 Neglected contribution of IT using LPI

essentially balanced to maintain the momentum conservation. Thus, the σ filter only affects a small term (IT) which decreases rapidly as the ϕ value increases and F_r approaches 1.0; and Eq. (2) is very closely approximated by Eq. (3). Figure 3 shows the omitted proportion of the inertial term (IT multiplied by $1-\sigma$) in the LPI technique as a function of F_r and ϕ for m=1 and m=5; Figure 3 also shows that $(1-\sigma)$ IT decreases as F_r exceeds unity.

Numerical Experiments

The computational errors for the LPI technique, which totally or partially omits the inertial terms, are considered as differences between the results of using the complete momentum equation (dynamic routing) and the results of using the LPI modified equation. Numerical experiments are performed to compare the results from both methods for a broad range of unsteady flow conditions.

A 64-km reach of a rectangular channel with width of 61 m is used for the numerical testing. Three channel slopes, 0.0038, 0.0095 and 0.0189, and Manning's n of 0.03, 0.045 and 0.055 are used. Various unsteady flow conditions are specified by using different inflow hydrograghs as the upstream boundary condition with peak discharge ranging from 2264 cms to 4000 cms and time of rise for the hydrographs ranging from 0.1 hour to 24 hour. An automatically generated loop rating is used as the downstream boundary condition.

Two kinds of errors are examined in the testing, they are:

$$E_{pk}(\%) = \max\,[(Q_{\max}(x)^{dyn} - Q_{\max}(x)^{lpi}) \times 100 / Q_{\max}(x)^{dyn}]; \quad (x=0-64\ km) \qquad (10)$$

$$E_{rms}(\%) = 100\,[(\sum_{t=t_j}^{t_j+N'} (Q(t)^{dyn} - Q(t)^{lpi}) / (N-1)]^{1/2} / \bar{Q}^{dyn} \quad (at\ x=32\ km) \qquad (11)$$

in which the superscript refers the results from either the dynamic or the LPI technique, $\bar{Q}$ is the average discharge. E_{pk} (%) is the maximum normalized error in the computed peak profiles, and E_{rms} (%) is the normalized root-mean-square (RMS) error in the computed hydrographs.

The analysis reveals that the most influential factor affecting the contribution of the inertial terms is the parameter ϕ. It is expected that the errors arising from partially or totally omitting the inertial terms are also directly related to some representative ϕ. Figure 4 shows that the maximum error in the peak profile is well related with the parameter, $\phi_0 = n^2 g^{3/2} y_m^{1/6} T_r / [\lambda^2 (y_m - y_0)]$ in which T_r is the time of the rise of the inflow hydrograph; y_0 and y_m are the initial and peak depth of the inflow and they can be determined from Manning's equation $y = (nQ/\lambda B\sqrt{S_0})^{0.6}$. The RMS error, $E_{rms}(\%)$ shown in Figure 5 are the results from only supercritical flows in which the inertial terms are totally omitted in the LPI method; and ϕ is determined according to average values of the rising limb of the hydrographs. It is interesting to notice that a line from Eq. (8) with $F_r = 1.10$ can be used to fit these results although Eq. (8) does not represent the actual error. A value of 5 for m in the σ function (Eq. (4)) is used for these tests.

All the tests show that the overall errors in using the LPI technique are very small (less than 2%) for most situations ($\phi > 10$) and less than 6% for special situations when $5 \leq \phi \leq 10$, which are only applicable for near instantaneous large dam-failure induced floods in channels of very flat bed slope.

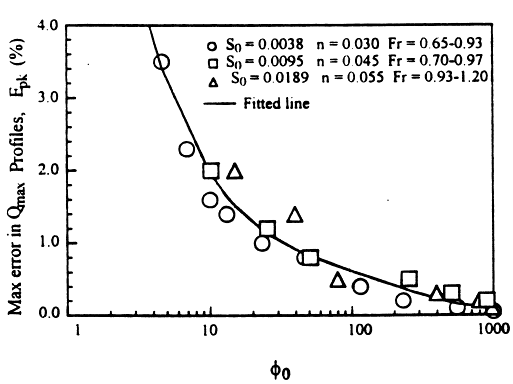

Figure 4 Errors in the computed peak flow

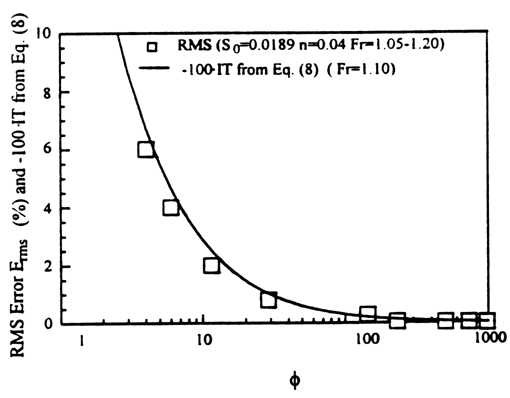

Figure 5 RMS errors in computed hydrographs

Example Application

The LPI enhanced FLDWAV model is applied to simulate a dam-break induced flood wave. The flood wave, with a peak discharge of about 2264 cms and a time of rise for the hydrograph of about 0.4 hours at the upstream boundary, travels through a 19.3-km reach of a extremely non-prismatic channel with a bottom slope varying from 0.0180 upstream to 0.0019 downstream and Manning's n varying from 0.05 upstream to 0.03 downstream. Mixed flow occurs along the steeper portions of the reach when the flow changes from the initial low flow to its peak, which causes numerical stability problems when using the conventional four-point implicit scheme.

Using the LPI technique, the FLDWAV model produces stable and smooth solutions for the flood wave simulation as shown in Figure 6. Also, Figure 6 compares the LPI computed hydrographs at four locations with: (1) those obtained from a characteristic-based upwind explicit dynamic routing technique which is available in the FLDWAV model to simulate nearly instantaneous dam-failure induced flood waves and near critical mixed flows; and (2) those from a previously developed mixed-flow algorithm (Fread, 1993) in the FLDWAV model. The explicit scheme has been tested successfully (Jin and Fread, 1995) for near instantaneous dam-break waves and mixed-flows. The close agreement between the more computationally efficient LPI technique and the explicit technique shows that the LPI produces reliable computational results whereas the mixed-flow technique is less accurate in simulating the near critical mixed flows with peak errors of 4-10%.

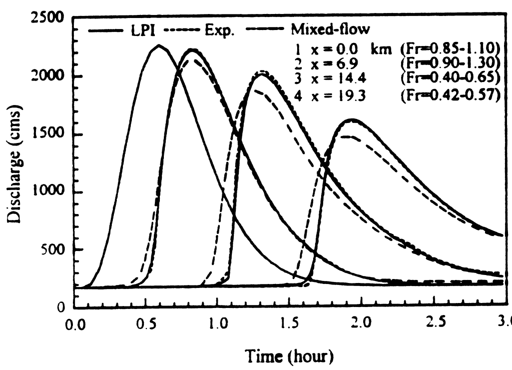

Figure 6 Computed dam-failure hydrographs

Conclusion

The LPI technique, which filters the inertial terms in the one-dimensional unsteady flow momentum equation according to the local Froude number, increases the FLDWAV model's stability in simulating near critical subcritical/supercritical mixed flows including supercritical/subcritical moving interfaces while retaining the accuracy of dynamic modeling for subcritical flows. The errors introduced by the LPI technique are found negligible,i.e, less than about 2% for most flow conditions including supercritical flows ($F_r > 1$), although errors can approach 4-6% for near instantaneous dam-failure induced floods propagating in channels of flat slope.

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